ELIZADE UNIVERSITY, ILARA MOKIN, NIGERIA

Mechanical Engineering Department

First Semester (2016/2017)

Course Code: MEE 407

Course Title: Advanced Mechanics of Materials

Time Allowed: 3 Hours

Instruction: Answer any four questions

1a). Differentiate between a Thin- and Thick-Walled Cylindrical Pressure Vessels.

b). Show that the circumferential stress in a cylindrical vessel is equal to twice the longitudinal stress

2. a) If $\sigma_r = \frac{E}{1-\nu^2}\left[C_1(1+\nu) - C_2\left(\frac{1-\nu}{r^2}\right)\right]$ and $\sigma_\theta = \frac{E}{1-\nu^2}\left[C_1(1+\nu) + C_2\left(\frac{1-\nu}{r^2}\right)\right]$,

show that the combined radial and tangential stresses is independent of radius

b). A steel shaft of 20 cm diameter is shrunk inside a bronze cylinder of 50 cm outer diameter. The shrink allowance is 1 part per 1000 (i.e. 0.01 cm difference between the radii). Find the circumferential stresses in the bronze cylinder at the inside and outer radii and the stress in the shaft.

$E_{steel} = 214 \times 10^6$ kPa

$E_{bronze} = 107 \times 10^6$ kPa

and $\nu = 0.3$ for both metals.

3a). Explain how a composite cylinder can support greater internal pressure than an ordinary one

b). Explain the concept of "Beams on an Elastic Foundation"

c). A flat steel disk of 85 cm outside diameter with a 15 cm diameter hole is shrunk around a solid steel shaft. The shrink-fit allowance is 1 part in 1000 (i.e. an allowance of 0.0085 cm in radius). E = 214×10^6 kPa)

4) Figure 1 represents an aluminum alloy I-beam (depth = 200 mm, $I_x = 2.45 \times 10^6$ mm^4, and E = 72.0 GPa) which has a length L = 7.0 m and is supported by seven identical springs (K = 110 N/mm) spaced at distance l = 1.15 m center to center along the beam. A load P = 13.0 kN is applied at the center of the beam over one of the springs.

a) Show that an approximate solution method will yield a reasonably good approximation

b) Using the approximate solution method, determine the maximum deflection of the beam under the load, the maximum bending moment, and the maximum bending stress in the beam.

c) If the deflection y_{max} occurs at the origin (at the center of the beam under the load), calculate the deflections of the springs at points C, B, and A

d) Compute the force R that each spring exerts on the beam

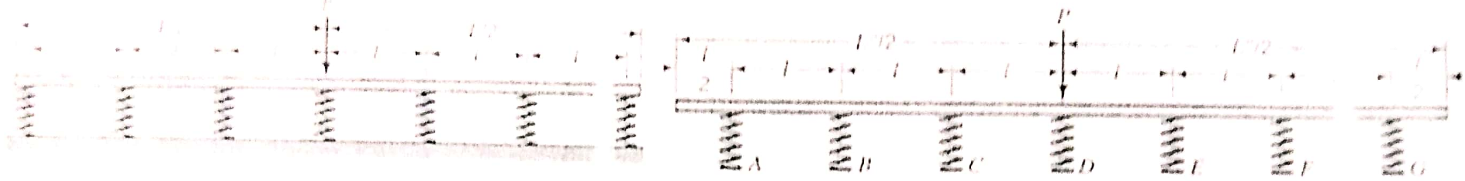

Figure 1: Aluminum Alloy I-Beam

5a). Explain what you understand by "Theory of Elasticity"

b). Show that the following displacement field cannot be a solution of a static plane stress problem

$u = d_1(x^2 - y^2) - d_2 y + d_3$ and $v = 2d_1 xy + d_4$ where the d_i are constants

6). A railroad uses steel rails (E = 200 GPa) with a depth of 174 mm. The distance from the top of the rail to its centroid is 99.7 mm, and the moment of inertia of the rail is 36.9 x 10^6 mm^4. The rail is supported by ties, ballast, and a road bed that together are assumed to act as an elastic foundation with spring constant k = 14.0 N/mm^2.

(a) Determine the maximum deflection, maximum bending moment, and maximum flexural stress in the rail for a single wheel load of 170 kN.

(b) A particular diesel locomotive has three wheels per bogie equally spaced at 1.70m. Determine the maximum deflection, maximum bending moment, and maximum flexural stress in the rail if the load on each wheel is 170 kN.

NOTE

$$\beta = \sqrt[4]{\frac{k}{4EI_x}} \qquad P_i = \frac{E\delta}{b}\left[\frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)}\right] \qquad P_i = \frac{\delta / b}{\frac{1}{E_1}\left[\frac{b^2 + a^2}{b^2 - a^2} - \nu_1\right] + \frac{1}{E_2}\left[\frac{c^2 + b^2}{c^2 - b^2} + \nu_2\right]}$$

Case 1: Internal Pressure only (P_o= 0):

$$\sigma_\theta = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right] \quad : \quad \sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right] \quad : \quad \sigma_z = \frac{P_i r_i^2}{r_o^2 - r_i^2}$$

At inside surface, r = r_i:

$$\sigma_\theta = P_i\left[\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right] \quad : \quad \sigma_r = -P_i \quad : \quad \sigma_z = \frac{P_i r_i^2}{r_o^2 - r_i^2}$$

At outside surface, r = r_o:

$$\sigma_\theta = \left[\frac{2P_i r_i^2}{r_o^2 - r_i^2}\right] \quad : \quad \sigma_r = 0 \quad : \quad \sigma_z = \frac{P_i r_i^2}{r_o^2 - r_i^2}$$

Case 2: External Pressure only (P_i= 0):

$$\sigma_\theta = \frac{-P_o r_o^2}{r_o^2 - r_i^2}\left[1 + \frac{r_i^2}{r^2}\right] \quad : \quad \sigma_r = \frac{-P_o r_o^2}{r_o^2 - r_i^2}\left[1 - \frac{r_i^2}{r^2}\right] \quad : \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2 - r_i^2}$$

At inside surface, r = r_i:

$$\sigma_\theta = \left[\frac{-2P_o r_o^2}{r_o^2 - r_i^2}\right] \quad : \quad \sigma_r = 0 \quad : \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2 - r_i^2}$$

At outside surface, r = r_o:

$$\sigma_\theta = -P_o\left[\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right] \quad : \quad \sigma_r = -P_o \quad : \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2 - r_i^2}$$

βz	$A_{\beta z}$	$B_{\beta z}$	$C_{\beta z}$	$D_{\beta z}$
0	1	0	1	1
0.02	0.9996	0.0196	0.9604	0.9800
0.04	0.9984	0.0384	0.9216	0.9600
0.10	0.9906	0.0903	0.8100	0.9003
0.20	0.9651	0.1627	0.6398	0.8024
0.30	0.9267	0.2189	0.4888	0.7078
0.40	0.8784	0.2610	0.3564	0.6174
0.50	0.8231	0.2908	0.2414	0.5323
0.60	0.7628	0.3099	0.1430	0.4529
0.70	0.6997	0.3199	0.0599	0.3798
$\frac{1}{4}\pi$	0.6448	0.3224	0	0.3224
0.80	0.6353	0.3223	−0.0093	0.3131
0.90	0.5712	0.3185	−0.0658	0.2527
1.00	0.5083	0.3096	−0.1109	0.1987
1.10	0.4476	0.2967	−0.1458	0.1509
1.20	0.3898	0.2807	−0.1716	0.1091
1.30	0.3355	0.2626	−0.1897	0.0729
1.40	0.2849	0.2430	−0.2011	0.0419
1.50	0.2384	0.2226	−0.2068	0.0158
$\frac{1}{2}\pi$	0.2079	0.2079	−0.2079	0
1.60	0.1960	0.2018	−0.2077	−0.0059
1.70	0.1576	0.1812	−0.2046	−0.0236
1.80	0.1234	0.1610	−0.1985	−0.0376
1.90	0.0932	0.1415	−0.1899	−0.0484
2.00	0.0667	0.1230	−0.1793	−0.0563
2.20	0.0244	0.0895	−0.1547	−0.0652
$\frac{3}{4}\pi$	0	0.0671	−0.1342	−0.0671
2.40	−0.0056	0.0613	−0.1282	−0.0669
2.60	−0.0254	0.0383	−0.1020	−0.0637
2.80	−0.0369	0.0204	−0.0777	−0.0573
3.00	−0.0422	0.0071	−0.0563	−0.0493
π	−0.0432	0	−0.0432	−0.0432
3.20	−0.0431	−0.0024	−0.0383	−0.0407
3.40	−0.0408	−0.0085	−0.0238	−0.0323
3.60	−0.0366	−0.0121	−0.0124	−0.0245
3.80	−0.0314	−0.0137	−0.0040	−0.0177
$\frac{5}{4}\pi$	−0.0278	−0.0140	0	−0.0139
4.00	−0.0258	−0.0139	0.0019	−0.0120
$\frac{3}{2}\pi$	−0.0090	−0.0090	0.0090	0
2π	0.0019	0	0.0019	0.0019